<table>
<tr><td>Alan Tupaj
Vista Murrieta High School
Website: www.vmhs.net
(Click on “Teachers” then “Alan Tupaj”)</td><td>U-Substitution
AP Readiness Session 5

Answers to examples posted on my website</td></tr>
<tr><td><u>U-Substitution Questions</u></td><td><u>Examples</u></td></tr>
<tr><td>Integrating a function to a power:

• Identity the inside function u
• Differentiate and isolate du
• Adjust for different or missing constant
• Substitute u and du
• Integrate resulting function using power rule
• Substitute back original function and add C</td><td>$\int x(x^2+3)^5dx$ $u = x^2+3$ $\frac{du}{dx} = 2x$ $du = 2xdx$
$\frac{1}{2}\int 2x(x^2+3)^5dx$ Need $2xdx$ inside integral
Multiply inside by 2 and outside by $\frac{1}{2}$
$\frac{1}{2}\int (u)^5du$ Substitute $u = x^2+3$, $du = 2xdx$
$= \frac{1}{2}\frac{u^6}{6}+C$ Integrate using power rule
$= \frac{(x^2+3)^6}{12}+C$ Substitute back for u</td></tr>
<tr><td>Integrating a trigonometric function (including powers on trig functions)

• Identify a function u and its derivative du
• Adjust for different or missing constant
• Substitute u and du
• Integrate as single trig function or using the power rule or integrate to directly to another trig function
• Substitute back original function and add C</td><td>A. $\int (\sin x)\cos^3(x)dx$ $u = \cos x$ $du = -\sin xdx$
$-\int (-\sin x)\cos^3(x)dx$ Need $-\sin x$
Multiply inside and outside by -1
$-\int u^3du$ Substitute $u = x^2+3$, $du = 2xdx$
$= -\frac{u^4}{4}+C$ Integrate using power rule
$= -\frac{\cos^4 x}{4}+C$ Substitute back for u

B. $\int (\sec(5x)\tan(5x)dx$ $u = 5x$ $du = 5dx$

(u is not a trig function since du would not exist in the integral)

$\frac{1}{5}\int \sec(5x)\tan(5x)(5)dx$
$\frac{1}{5}\int \sec(u)\tan(u)du$
$= \frac{1}{5}\sec(u)+C$ = $\frac{1}{5}\sec(5x)+C$</td></tr>
</table>

Integrating functions in denominators Careful: A power in the denominator is just a negative exponent, but a function without a power in the denominator will be integrated as *ln* • Identify a function *u* and its derivative *du* • Adjust for different or missing constant • Substitute *u* and *du* • Integrate with negative exponent or *ln* • Substitute back original function and add C	A. $\int \frac{(2x-1)dx}{x^2-x+5}$ $u = x^2 - x + 5$ $du = (2x-1)dx$ $\int \frac{du}{u}$ No constant adjustment needed $= \ln\|u\| + C$ Integrate using ln rule $= \ln\|x^2 - x + 5\| + C$ Substitute back for u B. $\int \frac{x^2 dx}{(x^3-4)^2}$ $u = x^3 - 4$ $du = 3x^2 dx$ $\frac{1}{3}\int \frac{3x^2 dx}{(x^3-4)^2}$ Adjust for constant $\frac{1}{3}\int \frac{du}{(u)^2}$ Substitute u and du $= \frac{1}{3}\left(\frac{u^{-1}}{-1}\right) + C = \frac{-1}{3(x^3-4)} + C$
Integrating functions that result in inverse trig functions • Factor to get the correct format (need a value of 1 in denominator) • Identify a function *u* and its derivative *du* • Adjust for different or missing constant • Substitute *u* and *du* • Integrate as an inverse trig function • Substitute back original function and add C	$\int \frac{dx}{4+16x^2}$ $u \neq 4 + 16x^2$ since $du = (32x)dx$ is not in the integral $= \frac{1}{4}\int \frac{dx}{1+4x^2}$ Factor out 4 to get the form of $\tan^{-1} u$ $= \frac{1}{4}\int \frac{dx}{1+(2x)^2}$ $u = 2x$ $du = 2dx$ $= \frac{1}{2} \bullet \frac{1}{4}\int \frac{2dx}{1+(2x)^2}$ Adjust for constant $= \frac{1}{8}\int \frac{du}{1+u^2}$ Substitute u and du $= \frac{1}{8}\tan^{-1}(u) + C = \frac{1}{8}\tan^{-1}(2x) + C$